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Models for light-cone meson distribution amplitudes

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Abstract: Leading-twist distribution amplitudes (DAs) of light mesons like π, ρ etc. describe the leading nonperturbative hadronic contributions to exclusive QCD reactions at large energy transfer, for instance electromagnetic form factors. They also enter B decay amplitudes described in QCD factorisation, in particular nonleptonic two-body decays. Being nonperturbative quantities, DAs cannot be calculated from first principles, but have to be described by models. Most models for DAs rely on a fixed order conformal expansion, which is strictly valid for large factorisation scales, but not always sufficient in phenomenological applications. We derive models for DAs that are valid to all orders in the conformal expansion and characterised by a small number of parameters which are related to experimental observables.

Keywords: B-Physics, Phenomenological Models, QCD.

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1. Introduction

Light-cone distribution amplitudes (DAs) appear in the description of hard exclusive QCD processes by factorisation, schematically

$$\text{amplitude} \sim \prod_{j,i} \phi_{\text{out},j}(n_j) \otimes T(Q^2; n_j, n_i) \otimes \phi_{\text{in},i}(n_i)\,, \tag{1.1}$$

where the labels i and j refer to the hadrons in the incoming and outgoing states, respectively, $\phi(n)$ is the DA that describes the probability amplitude for a hadron to be found in the parton state n and $T(Q^2; n_i, n_j)$ is the perturbative function that describes the hard scattering between the partons; Q^2 is the hard momentum transfer. The symbol $\otimes$ indicates a convolution, i.e. a sum or integral over the parton degrees of freedom that correspond to the states n_i and n_j. Factorisation implies that the process can be split into two separate regimes, the hard-scattering subprocess T, which involves a short-distance momentum transfer and can be calculated in perturbation theory, and the long-distance hadronisation of the partons emerging from the short-distance process, described by the process-independent DAs ϕ. The expansion of the amplitude in (1.1) is ordered by inverse powers of the momentum transfer Q^2, which corresponds to a light-cone expansion of the process in terms of contributions of increasing twist. In this paper we are concerned with the leading-twist DA of light mesons. Although we shall focus on the pseudoscalar mesons π and K, the models we suggest are equally well applicable to — and in fact have already [1] been applied to — vector mesons (ρ, ω, K^* and ϕ).

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The leading-twist meson DA of a meson is related to its Bethe-Salpeter wave function by integrating out the dependence on the transversal momentum $k_\perp$,

$$\phi(u) \sim \int_{k_\perp^2<\mu^2} d^2k_\perp \phi(u, k_\perp)\,,$$

where u is the longitudinal momentum fraction carried by the quark (and $\bar{u} \equiv 1-u$ that carried by the antiquark). Originally introduced in the context of hadron electromagnetic form factors or the pseudoscalar-photon transition form factor [2, 3], light-meson DAs have, in recent years, attracted increasing interest also in B physics due to their appearance in QCD sum rules on the light-cone [1, 4−6] and factorisation formulas for B decay amplitudes [7, 8] and form factors [9]. The theory of meson DAs is well understood [10−12] and suggests their parameterisation in terms of a partial wave expansion in conformal spin. One advantage of this expansion is that the contributions of higher conformal spins to the convolution integral and hence the physical amplitude are suppressed by the highly oscillating behaviour of the corresponding partial waves, another one that conformal symmetry ensures, to leading-logarithmic accuracy, multiplicative renormalisation of each partial wave. The combination of both features suggests the construction of models for DAs based on a truncated conformal expansion, where only the first few waves are included, typically one to three. Not much is known, in general, about the amplitudes of these partial waves. For the π and η some experimental information on the $\pi(\eta)\gamma\gamma^*$ transition form factor is available [13], which one can use to extract values of the first two amplitudes. From the theoretical side, there exist a few dated lattice calculations for the second moment of the π DA [14], and a recent retry [15]; unfortunately, these results are still preliminary and cannot yet be used in phenomenological applications. Other theoretical calculations, for both pseudoscalar [10, 11, 16, 17] and vector mesons [12, 18], use the method of QCD sum rules, which turns out to be not very suitable for higher moments; we will come back to that point in section 3.[1] It is probably fair to say that at present some information is available on the lowest moments of the π DA, but much less so for other mesons. In view of the growing demands on B physics to deliver "precision results", cf. e.g. ref. [21], and the prominent role of QCD factorisation in testing the mechanism(s) of flavour violation, it then appears timely to assess the actual theoretical uncertainty of hadronic decay amplitudes induced by the truncation of the conformal expansion and to devise alternative models for light-meson DAs that do not rely on conformal expansion, but establish a closer connection between the characteristics of DAs accessible in "classical" applications of pQCD in hard exclusive processes and their use in B physics. This is precisely what we aim to achieve in this paper.

Our models for the leading-twist DAs of the π and K are based on the fall-off behaviour of the nth Gegenbauer moment of these amplititudes, a_n, in n, which we assume to be power-like. We shall argue that such a behaviour can be justified for instance from the known perturbative contributions to the a_n. We formulate our models in terms of a few parameters, notably the first inverse moment of the DA, which, for the π, is directly

[1]These problems have been evaded in refs. [19, 20] in a modifed version of QCD sum rules using nonlocal condensates.

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related to experimental data, and the strength of the fall-off of the a_n. The models can be summed to all orders in the conformal expansion and parameterise the full DA at a certain low-energy scale. The evolution to higher scales is done by solving the Brodsky-Lepage evolution equation [2] numerically. Depending on the parameters, the models also predict a nonstandard behaviour of the DAs at the endpoints, which affects for instance the scaling, in m_b, of the $B \to$ light meson form factors at zero momentum transfer.

Our paper does *not* aim to give a fully-fledged analysis incorporating all available constraints on the π DA from low-energy experimental data as scrutinised in refs. [20, 22, 23]. Rather, we aim to work out the gross features of leading-twist DAs, which are likely to apply to all light pseudoscalar and vector mesons.

Our paper is organised as follows: in section 2 we define the leading-twist DA of pseudoscalar mesons and discuss its partial-wave expansion in conformal spin. In section 3 we derive models for the DA based on the fall-off behaviour of higher partial-waves and formulate constraints on the model parameters. We also discuss the evolution of the DAs under a change of scale. We illustrate the impact of nonstandard DAs for two quantities, the semileptonic form factor $f_+^{B\to\pi}(0)$, section 4, and the branching ratios and CP-asymmetries of $B \to \pi\pi$ decays, section 5. We summarise and conclude in section 6. The appendix contains details about how to solve the evolution equation.

2. Definitions and conformal expansion

Distribution amplitudes are defined in terms of matrix elements of nonlocal operators near the light-cone. For pseudoscalar mesons P in particular one has

$$\begin{aligned}\langle 0|\bar{q}_1(x)\gamma_\mu\gamma_5[x,-x]q_2(-x)|P\rangle &= i f_P p_\mu \int_0^1 du\, e^{i\xi px}\left[\phi_P(u) + \frac{1}{4}\, m_P^2 x^2 \mathbb{A}_P(u) + O(x^4)\right] + \\ &\quad + \frac{i}{2}\, f_P m_P^2\, \frac{1}{px}\, x_\mu \int_0^1 du\, e^{i\xi px}\, \left[\mathbb{B}_P(u) + O(x^2)\right], \end{aligned} \tag{2.1}$$

where $\xi = 2u-1$ and we use $[x,y]$ to denote the Wilson-line connecting the points x and y:

$$[x,y] = \mathrm{P}\exp\left[ig\int_0^1 dt\,(x-y)_\mu A^\mu(tx+(1-t)y)\right]. \tag{2.2}$$

In eq. (2.1), ϕ_P is the leading twist-2 DA, whereas $\mathbb{A}_P$ and $\mathbb{B}_P$ contain contributions from higher-twist operators. The corresponding definitions of vector meson DAs can be found in ref. [11].

The extraction of the leading behaviour of the matrix elements on the light-cone yields ultraviolet divergences, whose regularisation generates a nontrivial scale-dependence that can be described by renormalisation group methods [2, 3]. Conformal invariance of QCD allows one to express the DA in terms of a partial wave expansion, also called conformal expansion, in contributions from different conformal spin, which do not mix with each other under a change of scale. This is true to leading logarithmic accuracy, but no longer the case at higher order, as the underlying symmetry is anomalous. For the leading-twist DA $\phi(u)$,

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for both pseudoscalars and vectors, the conformal expansion is in terms of Gegenbauer polynomials $C_n^{3/2}$,

$$\phi(u,\mu^2) = 6u(1-u)\sum_{n=0}^{\infty} a_n(\mu^2) C_n^{3/2}(2u-1)\,. \tag{2.3}$$

The coefficients a_n, the so-called Gegenbauer moments, renormalise multiplicatively to leading logarithmic accuracy:

$$a_n(Q^2) = a_n(\mu^2)\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{(\gamma_0^{(n)}-\gamma_0^{(0)})/(2\beta_0)} \tag{2.4}$$

with $\beta_0 = 11-(2/3)n_f$. For pseudoscalar and longitudinally polarised vector mesons, the one-loop anomalous dimension is given by [24]

$$\gamma_0^{(n)} = 8C_F\left(\psi(n+2)+\gamma_E-\frac{3}{4}-\frac{1}{2(n+1)(n+2)}\right), \tag{2.5}$$

whereas for transversely polarised vector mesons one has

$$\gamma_0^{(n)} = 8C_F\left(\psi(n+2)+\gamma_E-\frac{3}{4}\right). \tag{2.6}$$

For π, ρ, ω, η, η' and ϕ, G-parity ensures that $a_{\rm odd}=0$ and that the DA is symmetric under $u\leftrightarrow 1-u$, whereas for K and K^* the nonzero values of $a_{\rm odd}$ induce an antisymmetric component of the DA. $a_0\equiv 1$ is fixed by normalisation,

$$\int_0^1 \phi(u,\mu^2) = 1\,,$$

whereas all other a_n are intrinsically nonperturbative quantities. As they do not mix under renormalisation, eq. (2.3) is well suited to construct models for ϕ: truncating the series after the first few terms, typically three, yields a parameterisation of the DA that is "stable" under a change of scale, except for the numerical values of a_n. Despite there being no small expansion parameter in the game, such a *truncated conformal expansion* is often — but not always — a meaningful approximation to the full DA, as we shall see below.

As the anomalous dimensions are positive (except for $\gamma_0^{(0)}=0$), the contributions of higher a_n get suppressed for large scales, and for $\mu^2\to\infty$ the DA approaches the so-called asymptotic DA

$$\phi_{\rm as}(u) = 6u(1-u)\,. \tag{2.7}$$

For many processes involving DAs, in particular in B physics, it is usually argued that a truncated conformal expansion be sufficient for the calculation of physical amplitudes as long as the perturbative scattering amplitude is "smooth" — the reason being the highly oscillatory behaviour of higher order Gegenbauer polynomials. It is actually instructive to quantify this statement, for example for the simplest, but phenomenologically very relevant case of one meson in the initial or final state, so that the convolution integral reads

$$I = \int_0^1 du\,\phi(u)T(u)\,, \tag{2.8}$$

where T is the perturbative scattering amplitude.

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We distinguish the following cases:

(i) T is nonsingular for $u \in [0,1]$;

(ii) T has an integrable singularity at one of the endpoints;

(iii) T contains a nonintegrable singularity at one of the endpoints.

As a typical example for case (i), consider $T(u) = \sqrt{u}$, which yields

$$\int_0^1 du\, \phi(u)\sqrt{u} = \sum_{n=0}^{\infty} \frac{(-1)^n 36(n+1)(n+2)}{(2n-1)(2n+1)(2n+3)(2n+5)(2n+7)}\, a_n\,.$$

This result implies a strong fall-off $\sim 1/n^3$ of the coefficients of higher Gegenbauer moments a_n: assuming $a_i \equiv 1$ for all i, already the first three terms in the sum account for 98.8% of the full amplitude. In reality the convergence will be even better as all existing evidence points at $|a_n| \ll 1$ for $n \geq 1$.

As an example for case (ii) choose $T(u) = \ln u$, which yields

$$\int_0^1 du\, \phi(u) \ln(u) = -\frac{5}{6}\, a_0 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+3)}\, 3a_n\,.$$

The singularity at $u = 0$ evidently worsens the convergence of the series; again assuming $a_i \equiv 1$, the first three terms now overshoot the true result by 35%. In order to approximate the full amplitude to within 5% one now has to include nine terms, but the convergence will again be better in practice, thanks to the fall-off of a_n in n.

Case (iii) is more complicated and depends on the asymptotic behaviour of the a_n. For $T(u) = 1/u$, for instance, one obtains

$$\int_0^1 du\, \phi(u)\, \frac{1}{u} = 3\sum_{n=0}^{\infty}(-1)^n a_n\,. \tag{2.9}$$

Here the amplitude is finite only if the a_n fall off sufficiently fast in n. For stronger endpoint divergences the coefficients multiplying a_n start to grow in n and for $T \sim 1/u^2$ the integral diverges, even for the asymptotic DA, which would indicate a breakdown of factorisation for that process.

The conclusion to be drawn from this discussion is that models for ϕ, based on a conformal expansion that is truncated after the first few terms, are indeed appropriate for cases (i) and (ii), but less so for case (iii). Convolutions with $T \sim 1/u$ are actually very relevant both in hard perturbative QCD reactions, e.g. $\gamma\gamma^* \to \pi$, and in B physics, e.g. $B \to \pi\pi$ [7, 8]. Given the different weight the a_n do have in different convoluted amplitudes, and the fact that their impact is highest in convolutions of type (iii), it appears not unreasonable to base a parameterisation of ϕ not on, say, a_2 and a_4, setting all $a_{n>4} = 0$, but rather on the "worst case scenario" of eq. (2.9), where all a_n enter with the highest possible weight factor. This is the basic idea of our models of leading-twist DAs we shall elaborate on in the next section.

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3. Models for ϕ_π and ϕ_K

There are basically two properties that any viable model for the leading-twist π DA ϕ_π must fulfill:

(a) in the limit of large energies, $\mu^2 \to \infty$, the DA must approach the asymptotic DA $\phi_\pi(u, \mu^2 = \infty) = 6u\bar{u}$ [2];

(b) the first inverse moment, $\int_0^1 du\, \phi_\pi(u)/u$, which is related to the $\pi\gamma\gamma^*$ transition form factor, must exist.

Condition (a) must evidently be fulfilled also for all other light-meson DAs; condition (b) should be fulfilled in general if QCD factorisation in B decays [7, 8] is to make sense. Obviously any model that is based on a truncated conformal expansions fulfills both constraints, and in addition predicts that for $u \to 0, 1$, $\phi \sim u(1-u)$, independently of the factorisation scale. We shall show that this prediction is, in general, not fulfilled for models that are not truncated at fixed order in the conformal expansion.

The starting point for our models is the relation between the first inverse moment of ϕ and the sum of all Gegenbauer moments:

$$\int_0^1 du\, \frac{\phi(u,\mu^2)}{3u} \equiv \Delta(\mu^2) = 1 + \sum_{n=1}^{\infty} (-1)^n a_n(\mu^2)\,. \tag{3.1}$$

As mentioned before, one has to distinguish between mesons for which $a_{\rm odd}$ vanish due to G-parity, such as the π, and strange mesons, for which the odd moments induce an antisymmetric part of the DA. It is the former case we shall study first. Available experimental data for the π, as summarised in ref. [23], point at a value of Δ around 1.1 at the scale $\mu \approx 1.2\,$GeV, which implies that the infinite sum be convergent. Hence, even at the comparatively low scale 1.2 GeV the a_{2n} must fall off fast enough in n. Assuming an asymptotic equal-sign behaviour for large-N, a power-like fall-off as $a_{2n} \sim 1/n^p$ with p slightly larger than 1 is one of the slowest possible fall-offs. It turns out that DAs defined by a power-like fall-off of the Gegenbauer moments can actually be summed explicitly: using the generating function of Gegenbauer polynomials,

$$f(\xi, t) = \frac{1}{(1 - 2\xi t + t^2)^{3/2}} = \sum_{n=0}^{\infty} C_n^{3/2}(\xi)\, t^n\,,$$

the DA with moments

$$a_n = \frac{1}{(n/b+1)^a}\,, \quad \text{for } n \text{ even}\,,$$

is given by

$$\tilde{\phi}^+_{a,b}(u) = \frac{3u\bar{u}}{\Gamma(a)} \int_0^1 dt(-\ln t)^{a-1} \left(f(2u-1, t^{1/b}) + f(2u-1, -t^{1/b})\right). \tag{3.2}$$

In the same way one obtains a DA with alternating-sign behaviour of the Gegenbauer moments,

$$a_n = \frac{(-1)^{n/2}}{(n/b+1)^a}\,, \quad \text{for } n \text{ even}\,,$$

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as

$$\tilde{\phi}^-_{a,b}(u) = \frac{3u\bar{u}}{\Gamma(a)}\int_0^1 dt(-\ln t)^{a-1}\left(f(2u-1,it^{1/b})+f(2u-1,-it^{1/b})\right). \tag{3.3}$$

The corresponding values of Δ are $\Delta^+_{a,b} = (b/2)^a\zeta(a,b/2)$ and $\Delta^-_{a,b} = (b/4)^a(\zeta(a,b/4) - \zeta(a,1/2+b/4))$, where $\zeta(a,s) = \sum_{k=0}^\infty 1/(k+s)^a$ is the Hurwitz ζ function. In order to obtain models for arbitrary values of Δ, we split off the asymptotic DA and write

$$\phi^\pm_{a,b}(\Delta) = 6u\bar{u} + \frac{\Delta-1}{\Delta^\pm_{a,b}-1}\left(\tilde{\phi}^\pm_{a,b}(u) - 6u\bar{u}\right), \quad \text{valid for } a\geq 1 \text{ and } b>0\,. \tag{3.4}$$

Is there any reason why the a_n should fall off as inverse powers in n — other than that the corresponding DAs can be expressed in closed form via eq. (3.4)? The answer to this question relies on the behaviour of $a_n(\mu^2)$ under a change of scale. Evidently the models are defined at one particular scale, for instance the hadronic scale $\mu\approx 1.2\,\mathrm{GeV}$. At larger scales Q^2, the a_n change according to eq. (2.4). For large-N, we have

$$\gamma_0^{(n)} \overset{n\to\infty}{\approx} 8C_F\ln n + O(1)$$

and

$$a_n(Q^2) \overset{n\to\infty}{\approx} \frac{1}{n^{4C_F/\beta_0\,\ln(1/L)}}\,a_n(\mu^2)$$

with $L = \alpha_s(Q^2)/\alpha_s(\mu^2)$. That is: perturbative leading-order scaling induces a power-like fall-off of the a_n, at least for large-N. We take this as indication that such a behaviour is indeed intrinsic to QCD. One more consequence of scaling is that for $Q^2\to\infty$, i.e. $L\to 0$, the suppression of higher a_n is power-like with a power that approaches infinity, so that

$$\tilde{\phi}^\pm(Q^2\to\infty) = \tilde{\phi}^\pm(a\to\infty) = 6u(1-u)\,.$$

Hence $\phi^\pm_{a,b}$ as defined in (3.4) approaches the asymptotic DA in this limit, which implies that condition (a) follows from (b), at least for this class of models.

Once the parameters Δ, a and b are specified at a certain reference scale, the evolution of the model DAs to different scales is determined by its evolution equation [2]

$$\mu^2\frac{\partial}{\partial\mu^2}\phi(u,\mu^2) = \int_0^1 dv\, V(u,v,\mu^2)\phi(v,\mu^2)\,, \tag{3.5}$$

where the kernel V is calculable in perturbation theory and to lowest order in α_s corresponds to single-gluon exchange between quark and antiquark; V is known to two-loop accuracy [25]. To leading-logarithmic accuracy, (3.5) can be solved numerically as described in the appendix. The change of the model DAs $\phi^\pm_{3,3}(1.2)$ is illustrated in figure 1, for an evolution from $\mu = 1.2\,\mathrm{GeV}$ to $\mu = m_b = 4.8\,\mathrm{GeV}$. Evidently our model DAs are not "form-invariant" under a change of scale in the sense that the model DA resulting from the evolution to larger scales is no longer described by $\phi^\pm_{a,b}(\Delta)$ with suitably chosen Δ, a and b. This situation reminds of that with parton distribution functions, where the parametrisation is fixed at a low scale, typically 2 GeV, but the parameters are actually fitted to experimental data obtained at a large variety of scales, cf. e.g. ref. [26]. In our

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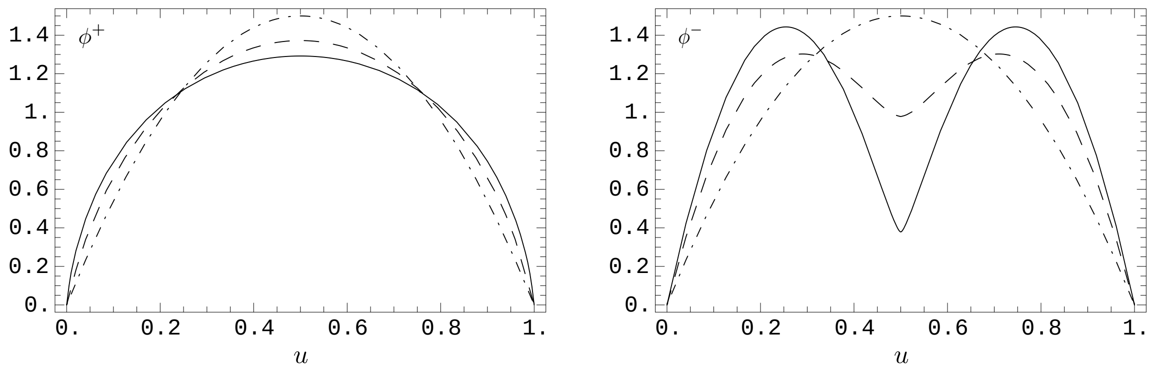

Figure 1: Evolution of $\phi^+_{3,3}(1.2)$ (left) and $\phi^-_{3,3}(1.2)$ (right) from $\mu = 1.2\,\mathrm{GeV}$ (solid curves) to $\mu = 4.8\,\mathrm{GeV}$ (dashed curves). For comparison we also show the asymptotic DA (dash-dotted curves).

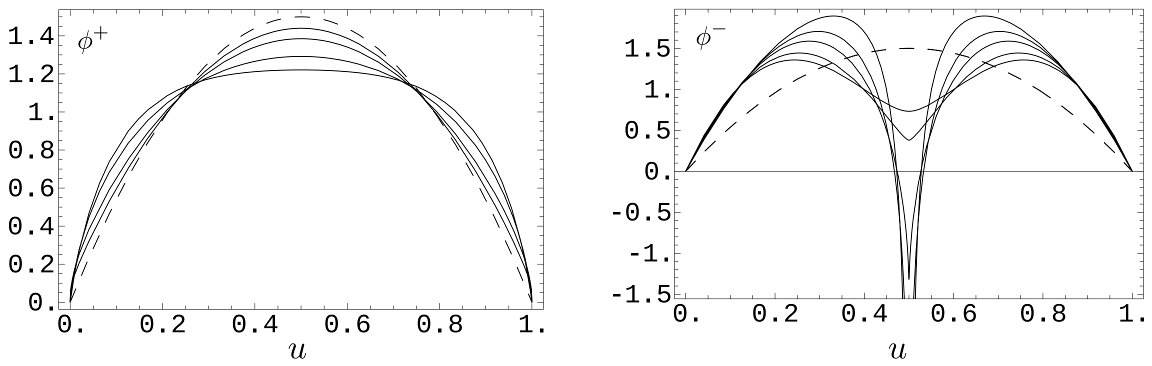

Figure 2: Left: Examples for model DAs $\phi^+_{a,b}$ as functions of u, for $a = 1.5, 2, 3, 4$, $b = 2$ and $\Delta = 1.2$ (solid curves), as compared to the asymptotic DA (dashed curve). For $a \to 1$, $\phi^+_{a,b}$ approaches the asymptotic DA. Right: the same for $\phi^-_{a,2}$. Note that for $a \leq 3$, $\phi^-_{a,b}$ is nonanalytic at $u = 1/2$ and displays a pronounced "spike".

case, the natural choice for the low scale is the scale at which Δ is determined, $\mu = 1.2\,\mathrm{GeV}$ according to the analysis of ref. [23]. The other two parameters a and b could ideally be fixed from experimental or theoretical (lattice, QCD sum rule) data obtained at the same or other scales, but as we shall see below the available constraints are presently not very stringent.

Eq. (3.4) implies that the asymptotic DA is recovered for $\Delta = 1$, and also from $\phi^+_{a,b}$ in the limit $a \to 1$. The models are valid only for $a \geq 1$, as otherwise $\Delta^+_{a,b}$ diverges, or, equivalently, $\phi^\pm_{a,b}$ does not vanish at the endpoints $u = 0, 1$. In figure 2 we plot several examples of $\phi^\pm_{a,b}$ for a fixed value of Δ; it is evident that the two models $\phi^+_{a,b}$ and $\phi^-_{a,b}$ have a rather dissimilar functional dependence on u; in particular $\phi^-_{a,b}$ turns out to be nonanalytic at $u = 1/2$ for $a \leq 3$. The dependence of $\phi^\pm_{a,b}$ on b is illustrated in figure 3. In figure 4 we show the possible values of the lowest Gegenbauer moments $a_{2,4}$ that can be obtained for fixed values of Δ and b, but different a. For $\phi^+_{a,b}$, one always has $a_2 < \Delta - 1$, for $\phi^-_{a,b}$, $a_2 > \Delta - 1$; in the limit $a \to \infty$ both branches meet and one obtains the truncated NLO conformal expansion with $a_2 = \Delta - 1$ and $a_4 = 0$.

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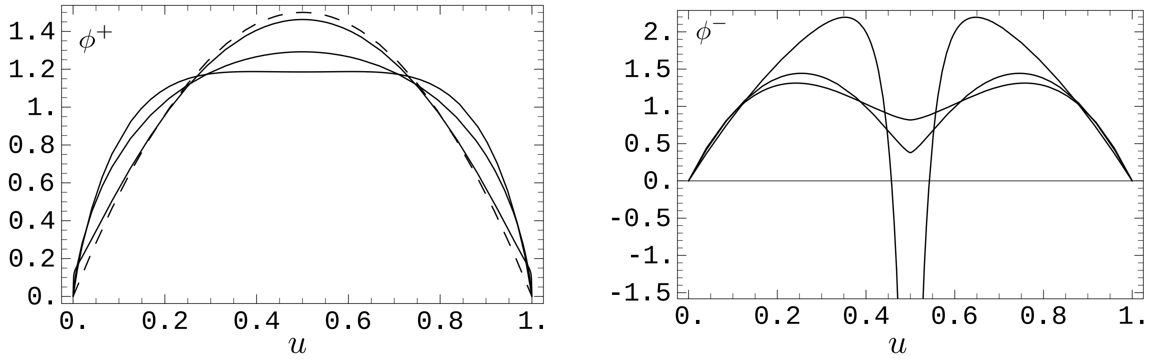

Figure 3: Left: Examples for model DAs $\phi^+_{3,b}$ as functions of u, for $b = 0.2, 2, 20$ and $\Delta = 1.2$ (solid curves), as compared to the asymptotic DA (dashed curve). For $b \to \infty$, $\phi^+_{a,b}$ approaches the asymptotic DA, as $\Delta^+_{a,\infty} \to \infty$. Right: the same for $\phi^-_{3,b}$.

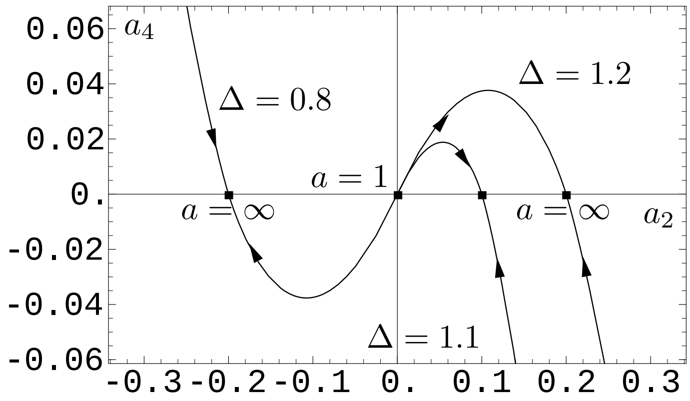

Figure 4: Values of a_2 and a_4 for different parameters Δ and a of $\phi^\pm_{a,2}$. The curves in the upper right and lower left quadrant correspond to $\phi^+_{a,2}$, those in the lower right and upper left one to $\phi^-_{a,2}$.

Before discussing constraints on the model parameters Δ, a and b, let us shortly comment on the behaviour of the DAs near the endpoints $u = 0, 1$. In many applications of DAs it is assumed — implicitly or explicitly — that $\phi \sim u(1-u)$ near the endpoints. Although this is evidently the case for any model based on a truncated conformal expansion, it does not apply to our models, at least not for all values of a. A closer inspection shows that $\phi^+ \sim \sqrt{u(1-u)}$ for $a = 2$ and $\sim u(1-u)\ln(u(1-u))$ for $a = 3$. It is only for $a > 3$ that the DAs behave in the "canonical" linear way. Are there are any rigorous arguments why the DAs should behave linearly near the endpoints, even for low hadronic scales? As far as we could trace the origin of the argument in favour of linear behaviour, it was stated first in ref. [10] and relies on the fact that QCD sum rules for the moments of the DA exhibit the following behaviour:

$$\langle \xi^n \rangle = \int_0^1 \xi^n \phi(u) \stackrel{n\to\infty}{\sim} \frac{1}{n^2}\,.$$

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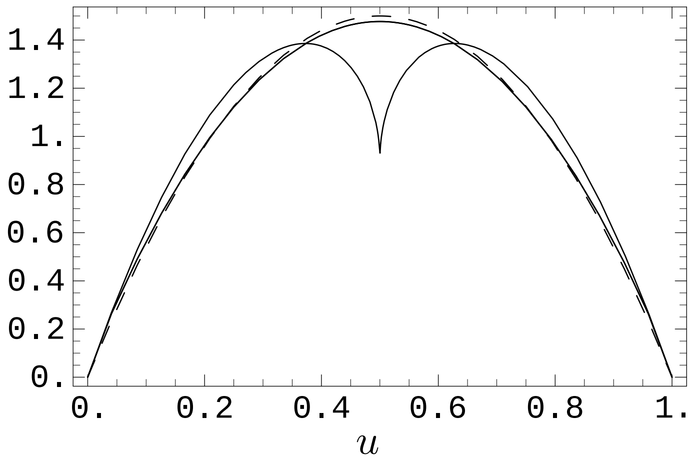

Figure 5: Comparison of $\phi^{\pm}_{2,2}(u)$ (solid curves) with the asymptotic DA (dashed curve) for $\Delta = 1.04$. The "spike" at $u = 1/2$ characteristic for ϕ^- causes these models to significantly deviate from the asymptotic DA even for Δ close to 1.

Two comments are in order here. First, an exact calculation for arbitary ϕ shows that

$$\langle \xi^n \rangle = \frac{1}{4n^2}\left(\phi'(0) - \phi'(1)\right) + O(n^{-3})\,,$$

so that the correct statement is that $\langle \xi^n \rangle \sim 1/n^2$ implies that $\phi'(u)$ exists at the endpoints — which in turn indeed implies a linear behaviour, since $\phi(0) = 0 = \phi(1)$ as long as $\Delta < \infty$. Second, the conclusion of the authors of [10] is based on results obtained from the leading-order perturbative contributions to QCD sum rules. NLO expressions have been obtained in [17, 18] and yield $\phi \sim u(1-u)\ln^2(u/(1-u))$, which upsets the linear approach to the endpoints and is equivalent to $a_n \sim 1/n^3$. The large-n behaviour of nonperturbative terms cannot be obtained from these sum rules,[2] but we see no reason why it should not follow the $a_n \sim 1/n^3$ behaviour of NLO perturbation theory or even introduce $1/n^2$ scaling. We hence conclude that the standard assumption of $\phi \sim u(1-u)$ is not rigorously justified at hadronic scales and that all conclusions relying on this feature have to be taken *cum grano salis.* We also would like to add one more remark about the expectation that for small $\Delta - 1$ the DA should be "very close" to the asymptotic DA. In figure 5 we show that this statement is indeed true for ϕ^+ models, but not for ϕ^-, which are characterised by their nonanalytic behaviour at $u = 1/2$. Of course the asymptotic DA is recovered for $\Delta - 1 \to 0$, but not in an analytic way.

Let us now discuss the experimental restrictions on Δ, a and b for the π. First, we fix the reference scale as $\mu = 1.2\,\mathrm{GeV}$. As the data are too scarce to constrain all parameters in a meaningful way, we fix $b \equiv 2$. As for Δ, we require it to be larger or equal 1. This is the case if $a_2 \geq 0$, which is indeed the common overall conclusion of all determinations available in the literature, as compiled in ref. [23]. An upper bound on Δ can be inferred

[2]The reason being that a_n for large-N are intrinsically nonlocal quantities, which cannot be obtained from a local operator product expansion (OPE). The ineligibility of the local OPE manifests itself as contributions to a_n that scale as *positive* powers in n, which is incompatible with a finite value of Δ.

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a	2	3	4	5	6	∞
$\Delta^+_{\rm max}$	2.04	1.58	1.43	1.36	1.33	1.27
$\Delta^-_{\rm max}$	1.04	1.11	1.16	1.19	1.22	1.27

Table 1: Upper bound on $\Delta^\pm$, for various values of a, as implied by $\phi_\pi(1/2) > 0.9$. The reference scale is $\mu \approx 1.2\,$GeV.

from experimental data or, for fixed a and b, from requiring $a_2 \leq 0.2$, which is again the maximum value allowed by most analyses.[3] For ϕ^+ the restriction on a_2 implies $\Delta \leq 1.2$, which coincides with the allowed range of Δ found in [20], whereas for ϕ^- the resulting range of Δ is smaller. One more constraint on $\phi^\pm$ is the allowed range of $\phi_\pi(1/2)$, which follows from light-cone sum rules for the πNN coupling [16]:

$$0.9 \leq \phi_\pi\left(\frac{1}{2}, 1\,\mathrm{GeV}\right) \leq 1.5\,.$$

For $\Delta > 1$, $\phi(1/2)$ is always smaller than 1.5, so it is only the lower bound that is relevant. In table 1 we list the corresponding maximum Δ for various a. Evidently the constraints on Δ^+ are weaker than the ones discussed before, but for ϕ^- the minimum value of $\phi_\pi(1/2)$ poses a nontrivial constraint on Δ.

It is possible to further refine the constraints for the π as it was done in e.g. [20, 22]. In this paper, however, we are not so much interested in the π, but rather in the gross characteristics of the DAs, which are likely to be valid also for other pseudoscalar and vector mesons. We hence refrain from pursuing that line of investigation, but summarise the main constraints on the symmetric parts of our model DAs, which are likely to be valid also for other mesons:

- $1 \leq \Delta \leq 1.2$ for $0 \leq a_2 \leq 0.2$ for $\phi^+_{a,2}$: this is based on the observation [10] that DAs of mesons with higher mass tend to become narrower;
- $1 \leq \mathrm{Min}(1.2, \Delta^-_{\rm max})$ for $\phi^-_{a,2}$, with $\Delta^-_{\rm max}$ given in table 1;
- $b = 2$, lacking further data.

Let us now turn to ϕ_K, the twist-2 DA of the K meson. It differs from ϕ_π by the fact that now also odd Gegenbauer moments contribute. Models for the antisymmetric part of the DA can be constructed in a similar way as before as[4]

$$\begin{aligned}
\tilde\psi^+_c(u) &= \frac{3u\bar u}{\Gamma(c)}\int_0^1 dt(-\ln t)^{c-1}\left(f(2u-1,\sqrt{t}) - f(2u-1,-\sqrt{t})\right),\\
\tilde\psi^-_c(u) &= \frac{3u\bar u}{i\Gamma(c)}\int_0^1 dt(-\ln t)^{c-1}\left(f(2u-1,i\sqrt{t}) - f(2u-1,-i\sqrt{t})\right). \qquad (3.6)
\end{aligned}$$

[3] With the notable exception of Chernyak and Zhitnitsky [10], whose results are however by now generally considered to be excluded by experiment.

[4] In complete analogy to the symmetric part of the DA we could introduce one more parameter d that would correspond to b. In view of the near complete absence of any information on antisymmetric DAs, we refrain from writing down the corresponding formulas and set $d \equiv 2$ from the very beginning.

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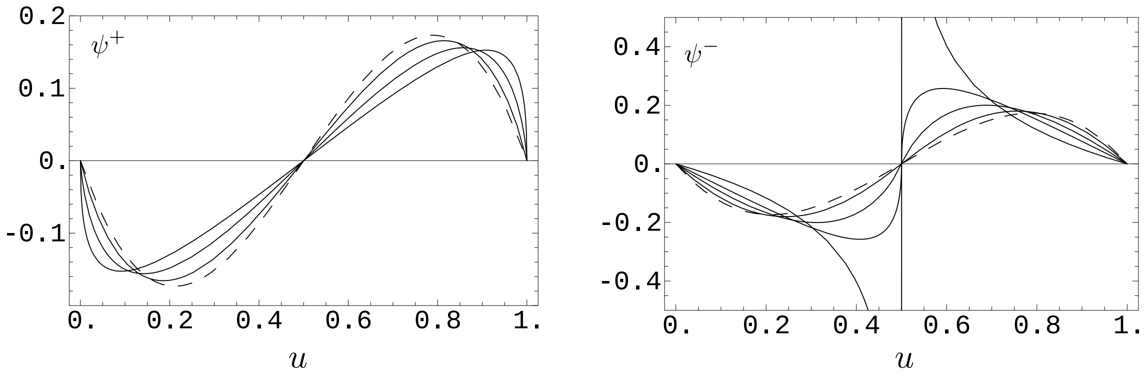

Figure 6: Models for the antisymmetric contributions to the twist-2 DA for $a_1 = 0.15$. Left: ψ_c^+ as function of u for $c \in \{2, 3, 5\}$ (solid curves), dashed curve: ψ_∞^+. Right: ψ_c^- as function of u for $c \in \{1, 2, 3, 5\}$ (solid curves), dashed curve: ψ_∞^-. Like the symmetric models ϕ_a^-, ψ_c^- is nonanalytic at $u = 1/2$ for $c \leq 3$.

The models are characterised by c and yield $a_1 = (2/3)^c$. Models for the antisymmetric part of ϕ with arbitrary a_1 can then be defined as

$$\psi_c^\pm = a_1 \left(\frac{3}{2}\right)^c \tilde{\psi}_c^\pm(u)\,. \tag{3.7}$$

Examples for such models are shown in figure 6. The antisymmetric part of the DA fulfilles the same evolution equation as the symmetric part and hence can be treated as discussed before for $\phi_{a,b}^\pm$.

Numerical values for a_1^K have been discussed in [6]; we quote $a_1^K(1.2\,\mathrm{GeV}) \approx 0.15$. For ϕ_K, the total $\Delta_{\rm tot}$ can then be written as

$$\Delta^{\rm tot,\pm} = \Delta + \Delta^{\rm asym,\pm}\,,$$

where $\Delta = \sum a_{\rm even}$ is the contribution of the symmetric part of the DA to $\Delta^{\rm tot}$ and $\Delta^{\rm asym,\pm}$ is defined as

$$\begin{aligned}
\Delta^{\rm asym,+} &= \int_0^1 du\, \frac{\psi_c^+(u)}{3u} = -a_1 \left(\frac{3}{2}\right)^c \zeta\left(c, \frac{3}{2}\right),\\
\Delta^{\rm asym,-} &= \int_0^1 du\, \frac{\psi_c^-(u)}{3u} = -a_1 \left(\frac{3}{4}\right)^c \left\{\zeta\left(c, \frac{3}{4}\right) - \zeta\left(c, \frac{5}{4}\right)\right\}.
\end{aligned}$$

4. Results for $f_+^{B\to\pi}(0)$

One important application of factorisation in B physics is the calculation of the weak decay form factor $f_+^{B\to\pi}$ from QCD sum rules on the light-cone. It is beyond the scope of this paper to review the method of QCD sum rules on the light-cone (LCSRs), for which we refer to ref. [4]. Instead, we would like to stress that LCSRs offer a means to calculate nonperturbative hadronic quantities like form factors within a controlled approximation, which relies on the factorisation of an unphysical correlation function, whose imaginary part is related to the hadronic quantity in question.

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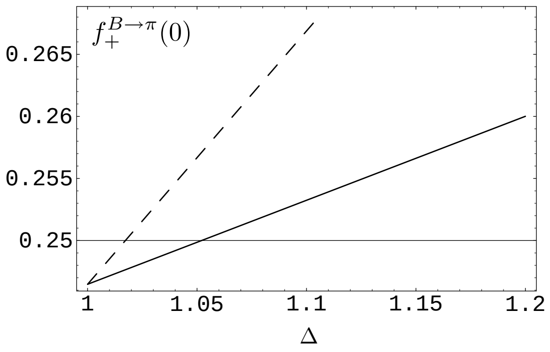

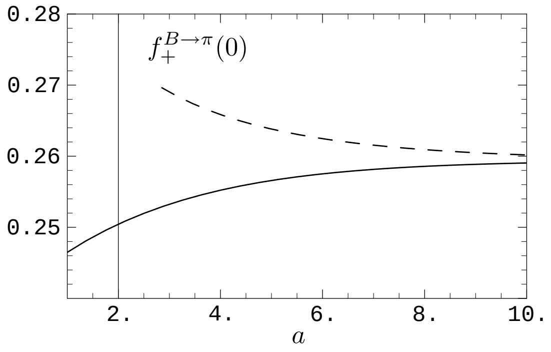

Figure 7: $f_+^{B\to\pi}(0)$ calculated from LCSRs, using the twist-2 DAs $\phi^+_{a,2}$ (solid lines) and $\phi^-_{a,2}$ (dashed lines). Left: $f_+^{B\to\pi}(0)$ as function of Δ for $a = 3$ fixed; right: $f_+^{B\to\pi}(0)$ as function of a for $\Delta = 1.1$ fixed. The endpoints of the dashed curves are set by the constraint $\phi(1/2) \geq 0.9$.

The $B \to \pi$ form factors are defined as

$$\langle\pi(p)|V_\mu|B(p_B)\rangle = \left[(p+p_B)_\mu - \frac{m_B^2 - m_\pi^2}{q^2}\, q_\mu\right] f_+^{B\to\pi}(q^2) + \frac{m_B^2 - m_\pi^2}{q^2}\, q_\mu\, f_0^{B\to\pi}(q^2)\,, \tag{4.1}$$

with $q = p_B - p$. Within the LCSR method $f_{+,0}^{B\to\pi}$ can be related to a correlation function

$$\begin{aligned}\Pi_\mu(q, p_B) &= i\int d^4x e^{iq\cdot x}\langle\pi(p)|TV_\mu(x) j_B^\dagger(0)|0\rangle \\ &= \Pi_+(q^2, p_B^2)(p+p_B)_\mu + \Pi_-(q^2, p_B^2) q_\mu\,,\end{aligned} \tag{4.2}$$

where $j_B = m_b \bar{d} i\gamma_5 b$ is the interpolating field for the B meson. For unphysical $p_B^2 \ll m_b^2$, $\Pi_\pm$ can be expanded around the light-cone as

$$\Pi_\pm^{\rm LC}(q^2, p_B^2) = \sum_n \int_0^1 du\, T_\pm^{(n)}(u, q^2, p_B^2, \mu)\phi^{(n)}(u,\mu)\,, \tag{4.3}$$

where the sum runs over contributions of increasing twist and the term in $n = 2$ is just the leading-twist contribution. $T_\pm^{(2,3)}$ are known to $O(\alpha_s)$ [6], whereas $T_\pm^{(4)}$ is known at tree-level [5]. In the most recent LCSR calculation of f_+, ref. [6], only contributions from a_2 and a_4 were included; the central values chosen for these parameters translate into $\Delta = 1.1$ and $a = 5.0$ (with $b = 2$ fixed). We have extended the calculation performed in [6] to include contributions up to a_{10}. The LCSR for the form factor f_+ depends on the spectral density $\rho_+^{\rm LC}$ of $\Pi_+^{\rm LC}$ in p_B^2 and reads

$$e^{-m_B^2/M^2} m_B^2 f_B\, f_+^{B\to\pi}(q^2) = \int_{m_b^2}^{s_0} ds\, e^{-s/M^2}\rho_+^{\rm LC}(s, q^2)\,, \tag{4.4}$$

where M^2, the so-called Borel parameter, and s_0, the continuum-threshold, are sum rule specific parameters. Using the central values of these parameters as obtained in [6], $M^2 = 9.2\,{\rm GeV}^2$ and $s_0 = 33.9\,{\rm GeV}^2$, and the model twist-2 DAs $\phi^\pm_{a,2}$ we obtain the values of $f_+^{B\to\pi}(0)$ shown in figure 7. The dependence of $f_+^{B\to\pi}(0)$ on Δ, with a fixed, is linear, as the values of the individual $a_{n\geq 2}$ are just rescaled by a common factor. If, on the

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other hand, Δ is kept fixed and a is being varied, higher a_n are increasingly suppressed with increasing a, so that for $a \to \infty$ the form factor obtained using ϕ^+ approaches that obtained using ϕ^-, as is clearly visible in figure 7.

Figure 7 shows that the theoretical uncertainty induced by Δ and a is about $\pm 5\%$. This has to be compared with the final value of $f_+^{B\to\pi}(0)$ stated in ref. [6]: 0.258 ± 0.031, i.e. a 12% uncertainty, which includes also the variation of other input parameters of the LCSR. The effect of our model DAs on this quantity is hence rather moderate and well accounted for by the original analysis of ref. [6], which is not completely unexpected as the LCSRs do not contain explicit terms of type Δ and hence there should not be a dramatic difference between using the full model DA or its truncated expansion.

5. $B \to \pi\pi$ in QCD factorisation

Another important application of leading-twist DAs are nonleptonic B decays treated in QCD factorisation [7, 8]. Recent experimental data point at a failure of QCD factorisation to explain the observed branching ratios and CP asymmetries in $B \to \pi\pi$ and $B \to \pi K$, which has prompted a number of authors to explain this effect by new physics, e.g. ref. [27], but has also motivated other authors to investigate the impact of nonfactorisable corrections to QCD factorisation, which are suppressed by inverse powers of m_b [28]. In this section we illustrate the effect of nonstandard DAs on the predictions of QCD factorisation for $B \to \pi\pi$. We use the formulas and central values of the input parameters as quoted in [8], except for the leading-twist π DA and the value of $f_+^{B\to\pi}(0)$ which we take from LCSRs, cf. the previous section.

In QCD factorisation the matrix elements determining the $B \to \pi\pi$ decay amplitude are written as a convolution of type [7, 8]:

$$\langle \pi\pi | Q_i | B \rangle = f_0^{B\to\pi}(0) T^I \star \phi_\pi + T^{II} \star \phi_B \star \phi_\pi \star \phi_\pi + O\left(\frac{1}{m_b}\right), \tag{5.1}$$

where Q_i is typically a four-quark operator, $f_0^{B\to\pi}(0) \equiv f_+^{B\to\pi}(0)$ is the semileptonic form factor discussed in the previous section, $T^{I,II}$ are perturbative kernels, ϕ_B is the B meson DA and ϕ_π the π DA. $T^{I,II}$ have been calculated to $O(\alpha_s)$ accuracy [7]. Part of the $1/m_b$ corrections have also been calculated; we include all factorisable corrections of this type, but not the unfactorisable ones (e.g. weak annihilation). In figure 8 we plot the ratio of the CP-averaged $B \to \pi\pi$ branching ratios calculated in QCD factorisation and the experimental results (HFAG [29]). The dashed lines denote the 1σ experimental error for $B \to \pi^0\pi^0$; those for the other decay channels are smaller. Evidently the branching ratios depend quite significantly on the shape of the π DA parametrised by Δ. In figure 9 we show the CP asymmetries of the neutral channels; C_{ab} is the direct CP asymmetry of $B \to \pi^a\pi^b$, S_{ab} the time-dependent asymmetry. As the figure illustrates, both S_{+-} and C_{+-} are largely independent of Δ, whereas C_{00} does depend on it.

Although the plots and results presented here do not include a fully-fledged error analysis and are hence mainly illustrative, they do suggest that the discrepancy between the experimental values of the branching ratios and their values predicted in QCD factorisation

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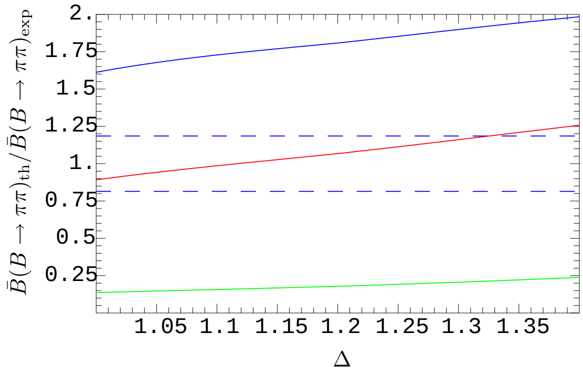

Figure 8: The ratio of CP-averaged branching ratios for $B \to \pi\pi$ decays in QCD factorisation as compared to the experimental result [29] as a function of Δ, for the model DAs $\phi^+_{3,2}$. $f_+^{B\to\pi}$ taken from LCSRs, all other input parameters as in [8]. Blue: $B^0 \to \pi^+\pi^-$, red: $B^+ \to \pi^+\pi^0$, green: $B^0 \to \pi^0\pi^0$; dashed lines: maximum exp. error (1σ).

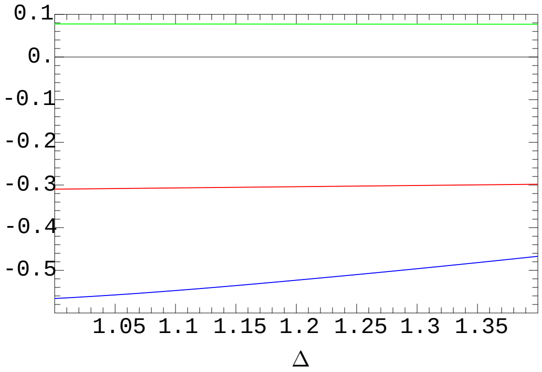

Figure 9: CP asymmetries in $B \to \pi\pi$ decays in QCD factorisation. Green: C_{+-}, red: S_{+-}, blue: C_{00}. We refrain from plotting experimental results as there are still discrepancies between BaBar and Belle.

can not be attributed to the uncertainties in the leading-twist DAs, not even for model DAs well outside the allowed parameter range. One possible explanation could be that power-suppressed nonfactorisable corrections are unexpectedly large as suggested in ref. [28]. A more detailed analysis of $B \to \pi\pi$ and $B \to K\pi$ decays in QCD factorisation and the size of nonfactorisable corrections will be presented elsewhere.

6. Summary and conclusions

We have presented models for the symmetric part of leading-twist light-cone distribution amplitudes (DAs) of light mesons, which depend on three parameters. Two of these parameters control the fall-off behaviour of the Gegenbauer moments a_n in n, whereas the third one, Δ, is given by the first inverse moment of the DA and parameterises the maximum possible impact of higher Gegenbauer moments on a factorised physical amplitude. Once the model DA is fixed at a certain low-energy scale, for instance $\mu = 1.2\,\mathrm{GeV}$ for the π, it can easily be evolved to higher scales by solving the evolution equation numerically. In contrast to the conformal expansion, which, for instance in MATHEMATICA, becomes numerically unstable at high orders, our model DAs are very well behaved numerically and can easily be implemented in computer codes. We have also developed similar models for the antisymmetric part of the DA, which is relevant for K and K^* mesons; these models are normalised to a_1, the first Gegenbauer moment. For the π DA, for which experimental data exist, we have formulated constraints on the model parameters which are likely to be valid also for other meson DAs. The constraints on the allowed values of Δ are tighter for the models ϕ^- with an alternating sign behaviour of the Gegenbauer moments than for those with an equal sign behaviour, ϕ^+. We feel that these models are better suited to

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estimate the hadronic uncertainty of processes calculated in factorisation than the standard truncated conformal expansion, in particular for processes that involve convolutions with singular kernels $\sim 1/u$, as the corresponding integral, Δ, is the basic parameter of the model.

We have illustrated the impact of the models on the calculation of the $B \to \pi$ weak decay form factor $f_+^{B\to\pi}(0)$ and the $B \to \pi\pi$ branching ratios and CP asymmetries. As for the form factor, the theoretical uncertainty induced by the model-dependence of the DA is smaller than that due to other parameters and approximations. For the nonleptonic decays we find that the branching ratios are more sensitive to the precise values of the model parameters than the CP-asymmetries. The observed discrepancy between experimental data and the predictions of QCD factorisation can, however, not be explained by nonstandard DAs, which indicates the presence of nonnegligible nonfactorisable contributions — a conclusion that agrees with the findings of other authors, e.g. refs. [28, 30].

Our models should prove particularly useful for describing DAs of mesons other than the π which are also symmetric by virtue of G-parity, but for which no experimental or other reliable theoretical information is available — in particular the ρ, ω and ϕ. For these particles, we argue that existing theoretical indications from local QCD sum rules [10, 18, 17] point at the DAs being narrower than that of the π, which implies the allowed values of Δ being smaller than 1.2 at the scale 1.2 GeV. The same results also imply a_2 to be positive, which entails $\Delta \geq 1$. For the parameter a, which controls the fall-off of the a_n in n, we have found that the perturbative contributions to QCD sum rules indicate it to be 3 [12], but that also smaller values of a are not excluded unless one can rigorously prove that the leading-twist DAs must behave as $\sim u(1-u)$ near the endpoints $u = 0, 1$, at *all scales*, which would imply $a \geq 4$. One more relevant constraint for the models $\phi^-_{a,b}$ with an alternating-sign fall-off of the Gegenbauer moments comes from the requirement $\phi_\pi(1/2) > 0.9$ at the scale $\mu = 1$ GeV. A positive value of this quantity is also required by QCD sum rules on the light-cone for the couplings $g_{DD^*\pi}$ [31] and $g_{DD^*\rho}$ [32], and hence also very likely to be the case for other mesons. Any further input, either from experiment or theory, in particular lattice, will be very welcome and help to constrain the model. Even without the constraints being taken too literally, our models provide a convenient way to test the impact of nonasymptotic DAs on physical amplitudes without recourse to conformal expansion.

Acknowledgments

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A. Leading-order evolution of $\phi^{\pm}_{a,b}$

To leading order in α_s, the kernel V in eq. (3.5) is given by [2]

$$V(x, y, \mu) = C_F \frac{\alpha_s(\mu)}{2\pi} V_0(x, y) + O(\alpha_s^2)$$

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with, for pseudoscalar mesons,

$$V_0(x,y) = V_{BL}(x,y) - \delta(x-y)\int_0^1 dz V_{BL}(z,y)\,,$$
$$V_{BL}(x,y) = \frac{1-x}{1-y}\left(1+\frac{1}{x-y}\right)\Theta(x-y) + \frac{x}{y}\left(1+\frac{1}{y-x}\right)\Theta(y-x)\,.$$

For vector mesons, one has two leading-twist DAs describing mesons with longitudinal and transverse polarisation, respectively, cf. the definitions in e.g. ref. [18]. The evolution equation of the longitudinal DA $\phi_\parallel$ coincides with that for pseudoscalar mesons, whereas for the transverse DA $\phi_\perp$ one has[5]

$$V_0^\perp(x,y) = V_{BL}^\perp(x,y) - \delta(x-y)\int_0^1 dz V_{BL}^\perp(z,y)\,,$$
$$V_{BL}^\perp(x,y) = \frac{1-x}{(1-y)(x-y)}\,\Theta(x-y) + \frac{x}{y(y-x)}\,\Theta(y-x)\,.$$

Eq. (3.5) is not very well suited for the numerical evolution of ϕ to different scales, because it requires the use of an explicit formula for α_s, which itself is the LO solution of a renormalisation group equation. In order to exactly reproduce the correct LO scaling behaviour of the Gegenbauer moments also numerically we rewrite (3.5) as a differential equation in α_s, which to LO reads

$$-\frac{\beta_0}{2C_F}\,\alpha_s\,\frac{\partial}{\partial\alpha_s}\,\phi(x,\alpha_s) = \int_0^1 dy\, V_0(x,y)\phi(y,\alpha_s)\,. \tag{A.1}$$

The above equation can be solved iteratively using Euler's method:

$$\phi(x,\alpha_s-\Delta\alpha_s) = \phi(x,\alpha_s) + \frac{2C_F}{\beta_0}\,\frac{\Delta\alpha_s}{\alpha_s}\int_0^1 dy V_0(x,y)\phi(y,\alpha_s)\,.$$

We have checked that for $\Delta\alpha_s = 0.01$ and with 21 mesh points in x we correctly reproduce the known scaling behaviour of the truncated conformal expansion to within one per mille. The inclusion of next-to-leading order effects is possible, but beyond the scope of this paper.

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